



2024 Academic Challenge

MATH TEST – REGIONAL

Math Production Team

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GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. One oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as  , not  ,  ,  , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

Time: 40 Minutes Number of Questions: 30

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

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Math (Regional) - 2024

1. A board game player is trying to roll doubles with two, six-sided dice. Given that the player's roll is at least a 9, what is the probability of getting doubles on that roll?
a) $1/2$ b) $1/6$ c) $1/5$ d) $1/3$ e) $2/3$
2. Which trinomial has two distinct, integral roots that are both prime numbers?
a) $x^2 + 7x + 8$ b) $2x^2 - 16x + 30$ c) $3x^2 + 6x - 9$ d) $2x^2 + 6x + 10$ e) $x^2 - 8x + 16$
3. If a circle is created by the equation $x^2 + y^2 = 64$, and another circle is centered at (2,2) and has a diameter of 8 units, what is the area of the region between the two circles?
a) 48π square units b) 192π square units c) 16 square units d) 24π square units
e) 16π square units
4. Which type of polygon cannot be inscribed in a circle? Assume the scaling of the polygon and the scaling of the circle are similar.
a) isosceles trapezoid b) right triangle c) scalene triangle
d) quadrilateral with angles of 30, 45, 120, and 165 e) rhombus with right angles
5. In the set of imaginary numbers, $i = \sqrt{-1}$. What is the value of i^{347} ?
a) 1 b) -1 c) i d) $-i$ e) 0
6. A square garden has an area of 625 square feet. If square tiles, each with an area of 1 square foot, are to be placed around the edges of the garden, how many tiles are needed?
a) 625 b) 125 c) 121 d) 100 e) 96
7. If the height of a rectangle is 5 inches, and the diagonal is 13 inches, what is the area of the rectangle?
a) 65 square inches b) 40 square inches c) 60 square inches
d) 32.5 square inches e) 45 square inches

8. In a system with the equations $2x + 3y = 12$ and $x + 4y = 8$, what ordered pair represents the intersection of the lines?

- a) (0.8, 4.8) b) (0.6, 4.4) c) (4.2, 0.6) d) (4.8, 0.8) e) The lines do not intersect.

9. In the conic section generated by the equation $(y^2/9) + (x^2/16) = 1$, what is the length of the major axis?

- a) 8 units b) 6 units c) 4 units d) 3 units e) 10 units

10. A sphere has a diameter of 12 inches. What is the volume of the sphere, to the nearest integer?

$$(V = 4/3(\pi)r^3)$$

- a) 402 cubic inches b) 15875 cubic inches c) 905 cubic inches
d) 8930 cubic inches e) 201 cubic inches

11. In the quadratic function $x^2 + 3x + 6$, what is the slope of the function at $x = 1$?

- a) 3 b) 5 c) 6 d) 10 e) 1

12. What are the discontinuities of the function $(x^2 - 2x - 15)/(x^2 - 4x - 5)$?

- a) Vertical asymptote at $x = 5$, vertical asymptote at $x = -1$.
b) Horizontal asymptote at $y = 0$, vertical asymptote at $x = 5$, vertical asymptote at $x = -1$.
c) Horizontal asymptote at $y = 1$, vertical asymptote at $x = 5$, vertical asymptote at $x = -1$.
d) Horizontal asymptote at $y = 0$, hole at $(5, 4/3)$.
e) Horizontal asymptote at $y = 1$, vertical asymptote at $x = -1$, hole at $(5, 4/3)$.

13. A list of data is composed of the numbers 1, 4, 6, 8, 8, 12, 17. What number can be added to the list without changing either the mean or the median?

- a) 0 b) 2 c) 6 d) 8 e) Not possible.

14. What is the least common denominator of the fractions

$$6/(x^2 - 7x - 18)$$

$$3/(x^2 + 8x + 12)$$

$$5/(x^2 - 12x + 27)$$

- a) $(x-9)(x+2)$ b) $(x+2)(x+6)(x-9)$ c) $(x-9)(x+2)(x+6)(x-3)$ d) $(x+2)(x+6)$
e) $(x+2)(x+6)(x-3)(x+9)$

15. In a right triangle, side AB is 8, side BC is 15, and side AC is 17. What is the tangent of angle A?

- a) $8/17$ b) $15/8$ c) $17/8$ d) $8/15$ e) $17/15$

16. What trigonometric value is undefined?

- a) $\sin \pi/3$
b) $\cos 2\pi/3$
c) $\tan 3\pi/2$
d) $\sec \pi$
e) $\csc \pi/2$

17. If the side measure of a cube is doubled, what happens to the surface area and the volume of the cube?

- a) Surface area increases by a factor of 2; volume increases by a factor of 8.
b) Surface area increases by a factor of 4; volume increases by a factor of 8.
c) Surface area increases by a factor of 4; volume increases by a factor of 6.
d) Surface area increases by a factor of 2; volume increases by a factor of 6.
e) Surface area increases by a factor of 8; volume increase by a factor of 16.

18. If only integers greater than 1 are considered, which set of integers always has an odd number of distinct, integral factors?

- a) Fibonacci numbers
- b) prime numbers
- c) odd numbers
- d) perfect squares
- e) even numbers

19. What is the value of $\log_9 3 - \log 0.1$?

- a) $-3/2$
- b) $-1/2$
- c) 0
- d) $1/2$
- e) $3/2$

20. Which linear equation generates a line that passes through the highest points of the function $1/2 (\cos 3(x+6)) + 4$?

- a) $y = 1$
- b) $y = 2$
- c) $y = 4.5$
- d) $y = 4$
- e) $y = 6$

21. A basketball player is shooting toward a regulation 10-foot basket. If the player is allowed to take steps back as needed, which equation would generate a shooting arc that would allow the ball to reach the basket?

- a) $y = -x^2 + 2x + 6$
- b) $y = -x^2 + 4x + 6$
- c) $y = -x^2 + 4x + 5$
- d) $y = -2x^2 + 8$
- e) $y = -2x^2 + 4x + 6$

22. The Richter scale of the magnitude of an earthquake is measured using logarithmic comparisons. If an earthquake measures 5.2 on the Richter scale, what is the magnitude of an earthquake that is 100 times more powerful?

- a) 5.40
- b) 5.22
- c) 5.90
- d) 6.22
- e) 7.20

23. Fahrenheit temperatures are converted to Celsius by subtracting 32 and then multiplying by $5/9$. If a set of Fahrenheit temperatures has a standard deviation of 4.5, what would the standard deviation be after converting the set to Celsius?

- a) -15.28
- b) -27.5
- c) -29.5
- d) 2.5
- e) 20.28

24. What is the probability of drawing two hearts from a standard deck of 52 cards if both cards are drawn without replacement?

- a) $1/169$ b) $2/169$ c) $1/16$ d) $1/12$ e) $1/17$

25. A cone has a radius of 4.5 meters and a height of 18 meters. If water is added to the cone, and the water level reaches a height of 10 meters, what is the volume of the water in the cone at that point? The equation for volume of a cone is $(1/3)\pi r^2 h$.

- a) 381.70 cubic meters b) 65.45 cubic meters c) 128.28 cubic meters d) 212.06 cubic meters
e) 94.25 cubic meters

26. If y varies inversely with x , and $y = 5$ when $x = 3$, calculate the value of y when $x = 10$.

- a) $1/3$ b) $2/3$ c) $10/3$ d) $3/2$ e) $1/2$

27. If a car is traveling 60 miles per hour, what is its speed in feet per second? (There are 5,280 feet in a mile.)

- a) 0.114 feet/second b) 1.47 feet/second c) 14.67 feet/second d) 88 feet/second
e) 112 feet/second

28. Determine the number of unique arrangements of the letters in the word LEVERAGE.

- a) 40,320 b) 13,440 c) 6,720 d) 512 e) 64

29. Along the unit circle, with a domain of $[0, 2\pi)$, how many locations have an equal value for tangent and cotangent?

- a) 0 b) 1 c) 2 d) 4 e) 8

30. A city is losing 2 percent of its population each year. If it has 63,000 people in 2023, how many people will it have in 2038? Round to the nearest integer.

- a) 44,100 b) 46,530 c) 50,400 d) 53,550 e) 55,250

Scratch Paper